STA362: Exam 1 (Fall 2001) **Name** Key

Instructions: *You must show all work to receive full credit, except in cases where calculator stat functions are used. Be sure your answers are in the context of the problems. Clearly articulate your written responses. Completeness and conciseness will benefit you. Point values in <>.*

1. The manufacturer of a power supply is interested in the variability of output voltage. He has tested 12 units, chosen at random, with the following results:

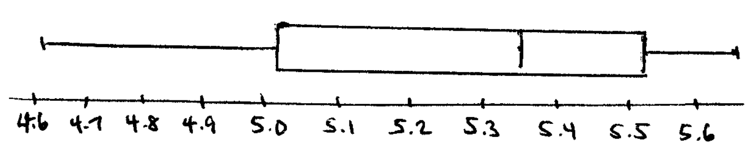
5.34	5.65	4.76	5.00	5.55	5.54
5.07	5.35	5.44	5.25	5.35	4.61

a] Find the sample mean and median for these data. <5+5>

$$\bar{X} = 5.2425 \text{ volts}$$

$$\tilde{X} = \frac{5.34 + 5.35}{2} = 5.345 \text{ volts}$$

b] Given that the lower quartile of this sample is 5.0175 and the upper quartile is 5.515, build an *accurate* box-and-whisker plot of this sample. From the plot, comment briefly on the shape of the distribution. <4+4>



This distribution appears to be skewed to the left, due to its asymmetric shape

c] Find the sample standard deviation. <6>

$$S = 0.3222 \text{ volts}$$

d] Suppose the voltage meter used to take the above measurements had been miscalibrated, and it was discovered that all the measurements were actually 0.785 higher than the readings indicated. What would be the sample mean and standard deviation for the "corrected" data? Offer a brief explanation as to why these "corrected" results relate as they do to the "miscalibrated" results. <4+4>

All observations would shift upward in location by 0.785; but their variability pattern will not change. So,

$$\bar{X}_{\text{corrected}} = 6.0275; \qquad S_{\text{corrected}} = 0.3222$$

2. Consider a sample of size $n = 20$ from a population with mean μ and standard deviation σ. We are interested in the precision (i.e., variability) of $\bar{X}$ in order to assess how accurately it will estimate μ. Suppose we wish to double the precision of this $\bar{X}$ estimate, i.e., we want to cut its standard deviation in half. What new sample size would be required to do so? <6>

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sigma}{\sqrt{20}}.$$

In order to get an $\bar{x}$ with twice the precision, we'd need

$$\sigma_{\bar{x}_{new}} = \frac{1}{2}\frac{\sigma}{\sqrt{20}} = \frac{\sigma}{\sqrt{4}\sqrt{20}} = \frac{\sigma}{\sqrt{80}}.$$

The new sample size would be $n = 80$.

3. The U.S. Army Engineering and Housing Support Center recently sponsored a study of the reliability, availability, and maintainability (RAM) characteristics of small diesel and gas-powered systems at commercial and military facilities. The study revealed that the time, X, to perform corrective maintenance on continuous diesel auxiliary systems has an approximate exponential distribution with an estimated mean of $\mu = 1{,}700$ hours. (Recall that exponential distributions have the unique property that $\mu = \sigma$.)

a] Based on this, what is the sampling distribution for $\bar{X}$ based on a sample of 70 continuous diesel auxiliary systems? [Cite values of the distribution's parameters, too.] <6>

From CLT, $\bar{X}$ is approximately normal w/ $\mu_{\bar{X}} = 1700$

$$\sigma_{\bar{X}} = \frac{1700}{\sqrt{70}} = 203.2$$

b] Assuming that $\mu = 1{,}700$, find the probability that the mean time to perform corrective maintenance for a sample of 70 continuous diesel auxiliary systems exceeds 2,500 hours. <10>

$$P(\bar{X} > 2500) = P\left(\frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} > \frac{2500 - 1700}{203.2}\right)$$
$$= P(Z > 3.94)$$
$$\leq 1 - .9998$$
$$< .0002.$$

c] If you in fact observe $\bar{X} > 2{,}500$, what inference would you make about the value of μ, and why? <6>

Since seeing $\bar{X} > 2500$ is so unlikely assuming that μ is really 1700, I would infer that μ is actually larger than 1700 ... and our $\bar{X}$ is a reflection of that.

d] Find the probability of getting a sample mean estimate $\bar{X}$ within $\pm$ 400 hours of the true mean μ. <10>

$$P(|\bar{X} - \mu| \leq 400) = P(-400 \leq \bar{X} - \mu_{\bar{X}} \leq 400)$$
$$= P\left(\frac{-400}{203.2} \leq \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} \leq \frac{400}{203.2}\right)$$
$$= P(-1.97 \leq Z \leq 1.97)$$
$$= .9756 - .0244 = .9512.$$

e] Find the probability of getting a sample standard deviation estimate s within $\pm$ 400 hours of the true standard deviation σ. [Think for a moment about this one.] <10>

We cannot use the χ^2-distribution here, because the pop'n is exponential, not normal. However, since in an exponential $\mu = \sigma$, it is true that $\bar{X}$ and s estimate the same thing; so, $P(|s - \sigma| \leq 400) = P(|\bar{X} - \mu| \leq 400) = .9512$ (from d).

4. Suppose that the television picture tubes of manufacturer A have a true mean lifetime of 6.5 years and a standard deviation of 0.9 year, while those of manufacturer B have a true mean lifetime of 6.4 years and a standard deviation of 0.8 year. A random sample of 36 tubes is chosen from manufacturer A's output, and a random sample of 27 tubes is chosen from manufacturer B's output.

What is the probability that, just by chance alone, the sample means <u>mistakenly</u> indicate that manufacturer B's picture tubes last <u>longer</u>; i.e., what is $P(\bar{x}_A - \bar{x}_B < 0)$? <10>

$\bar{x}_A - \bar{x}_B$ is approx normal; $\mu_{\bar{x}_A-\bar{x}_B} = 6.5 - 6.4 = 0.1$

$$\sigma_{\bar{x}_A-\bar{x}_B} = \sqrt{\frac{(0.9)^2}{36} + \frac{(0.8)^2}{27}} = 0.215$$

$$P(\bar{x}_A - \bar{x}_B < 0) = P\left(\frac{(\bar{x}_A-\bar{x}_B)-(\mu_A-\mu_B)}{\sqrt{\frac{\sigma_A^2}{n_A}+\frac{\sigma_B^2}{n_B}}} < \frac{0-0.1}{0.215}\right)$$

$$= P(z < -0.46)$$

$$= \underline{\underline{0.3228}}.$$

5. Suppose that the thickness of a part used in a semiconductor is its critical dimension, and that the process of manufacturing these parts is considered to be under control if the true variation among the thicknesses of the parts is given by a standard deviation no greater than $\sigma = 0.60$ thousandths of an inch. To keep a check on the process, random samples of size $n = 20$ are taken periodically, and it is regarded to be "out of control" if the probability that S^2 will take on a value greater than or equal to the observed sample value is 0.01 or less (even though $\sigma = 0.60$).

What can one assume about the process if the standard deviation of such a periodic random sample is $s = 0.84$ thousandth of an inch? <10>

The process will be declared 'out of control' if $P(\chi^2 > \chi^2_{.01}) = .01$; i.e., if $\frac{(n-1)s^2}{\sigma^2}$ with $n=20$ and $\sigma = 0.60$ exceeds $\chi^2_{.01,\,df=19} = 36.191$.

If $s = 0.84$, then $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{19(.84)^2}{(0.60)^2} = \underline{\underline{37.24}}$.

So, since when $s = 0.84$, we see $\chi^2 > 36.191$, <u>we would declare the process out of control.</u> (It must be assumed that we had a r. sample from a normal pop'n).