

STA362: Exam 3 (Fall 2001)

Name Key

**Instructions:** You must show all work to receive full credit, except in cases where calculator stat functions are used. Be sure your answers are in the context of the problems. Clearly articulate your written responses. Completeness and conciseness will benefit you. Point values in <>.

1. You are considering taking a job with a computer software firm. The CEO is considered a brilliant visionary by some people but is considered an ogre who abuses his new employees by others. You are trying to decide whether to accept a job offer from this firm. Suppose your hypotheses are

$H_0$ : the CEO is an ogre who will make my life miserable

$H_1$ : this is my dream job working at the cutting edge of software development with the most gifted person in the business.

- a) Explain, using terms any lay person could understand, what the implications are for you if you commit a type I error. <5>

reject  $H_0$  when  $H_0$  true  $\Rightarrow$  you believe that this is your dream job (so you take a job there) when in actuality the CEO is an ogre  $\Rightarrow$  your life is miserable

- b) Explain the implications of committing a type II error. <5>

accept  $H_0$  when  $H_0$  false  $\Rightarrow$  you miss the opportunity to have your dream job b'c you mistakenly believe the CEO is an ogre.

2. 120 Miami University Oxford undergraduates are going to be surveyed as to whether or not they feel that racial prejudice is a problem on campus. If fewer than 50 of the 120 surveyed (i.e., fewer than 41.7% of the sample) feel that racial prejudice is a problem on campus, it will be concluded that this is the minority opinion amongst all Miami University Oxford undergraduates (i.e., it will be concluded that  $p < 0.50$ .)

- a) What is  $\alpha$ , the probability of committing a type I error, if this decision rule is used? (First set up  $H_0$  and  $H_1$ .) <5>

$$\begin{aligned} \alpha &= P(\hat{p} < .417 \text{ when in fact } p = .5) \\ &= P\left(\frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} < \frac{.417 - .5}{\sqrt{\frac{.5(.5)}{120}}}\right) = P(Z < -1.82) \\ &= \underline{\underline{.0344}} \end{aligned}$$

- b) If, in truth, only 40% of all Miami University Oxford undergraduates feel that racial prejudice is a problem on campus, then what is the probability that this decision rule will mistakenly fail to conclude that  $p < 0.50$ ? <5>

$$\begin{aligned} \beta &= P(\hat{p} \geq .417 \text{ when } p = .4) \\ &= P\left(\frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} \geq \frac{.417 - .4}{\sqrt{\frac{.4(.6)}{120}}}\right) = P(Z \geq 0.38) \\ &= \cancel{0.3520} \\ &= \underline{\underline{.352}} \end{aligned}$$

3. Refer to the MU opinion survey from problem 2. We now go out and collect our 120 opinions at random on the Oxford campus. The collected data are below:

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a) Can you confidently state that those who feel that racial prejudice is a problem on campus constitute only a minority of the MU undergraduate population? Run a hypothesis test using the decision rule given in problem 2 to address this. (Show all elements of the testing procedure, including the p-value.) <20>

$H_0: p = .5$   
 $H_1: p < .5$

$TS: x = 45 \Rightarrow \hat{p} = \frac{45}{120} = .375 \quad \left( z = \frac{.375 - .5}{\sqrt{\frac{.5(.5)}{120}}} = -2.74 \right)$

$CR: \text{Reject } H_0 \text{ if } \hat{p} < .417$  (from problem 2). Since this is true,

REJECT  $H_0$  Yes, those who feel prejudice is a problem at MU constitute a minority.

$p\text{-value} = P(\hat{p} \leq .375) = P\left(\frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} \leq \frac{.375 - .5}{\sqrt{\frac{.5(.5)}{120}}}\right) = P(z \leq -2.74) = \underline{\underline{0.0031}}$

b) Suppose the same survey was given to a sample of 186 Ohio University students regarding racial prejudice on the OU campus. In that sample, 71 students felt racial prejudice is a problem there. Is there is reason to believe that the proportion of OU students that feel racial prejudice is a problem at OU differs from the proportion of MU students who feel the same way about Oxford? Run a hypothesis test using  $\alpha = 0.05$  to address this. (Show all elements of the testing procedure, including both a critical region and a p-value.) <20>

$H_0: p_{MU} = p_{OU}$   
 $H_1: p_{MU} \neq p_{OU}$

$\hat{p}_{MU} = \frac{45}{120} = .375$   
 $\hat{p}_{OU} = \frac{71}{186} = .382$   
 $\hat{p} = \frac{45 + 71}{120 + 186} = .379$

$TS: z = \frac{(.382 - .375) - 0}{\sqrt{.379(.621)\left(\frac{1}{120} + \frac{1}{186}\right)}} = \underline{\underline{0.12}}$

$CR: \text{Since } \alpha = .05, \text{ reject } H_0 \text{ if } z > 1.96 \text{ or } z < -1.96$

Conclusion: Fail to reject  $H_0$ . There is not enough evidence to believe that the prop. of OU students who feel prejudice is a problem at OU differs from the prop of MU students who feel that way about MU.

$p\text{-value} = 2 \cdot P(z > 0.12) = 2(-.4522) = \underline{\underline{.9044}}$

$$s_p = \sqrt{\frac{56(9.95)^2 + 34(13.62)^2}{90}} = 11.475$$

4. Data were collected on 57 males and 35 females that were chosen randomly; suppose the samples were chosen independently from one another. Summary statistics on sample pulse rates are shown below.

	N	MEAN	MEDIAN	SPDEV	MIN	MAX	Q1	Q3
M_pulse	57	70.42	70.00	9.95	48.00	92.00	63.00	75.00
F_pulse	35	76.86	78.00	13.62	58.00	100.00	66.00	86.00

An analyst looking at stem and leaf plots of the male and female pulse rates decided that it was reasonable to assume the data came from normal populations. Is there convincing evidence that the true mean pulse rate is higher for females than for males? Run a hypothesis test using  $\alpha = 0.05$  to address this. (Show all elements of the testing procedure, including both a critical region and a  $p$ -value.) <20>

$$H_0: \mu_F = \mu_M$$

$$H_1: \mu_F > \mu_M$$

$$TS: t = \frac{(\bar{x}_F - \bar{x}_M) - 0}{s_p \sqrt{\frac{1}{n_F} + \frac{1}{n_M}}} = \frac{76.86 - 70.42}{11.475 \sqrt{\frac{1}{57} + \frac{1}{35}}} = 2.613 \quad (df = 90)$$

$$CR: (\alpha = .05, df = 60 - \text{table}): \text{reject } H_0 \text{ if } t > 1.671$$

Reject  $H_0$ . Yes, the true mean pulse rate is higher for females than for males.

$$p\text{-value} = P(t > 2.613) \rightarrow .005 < p\text{-value} < .0075$$

5. A manufacturer suspects that the quality of items shipped to them for a production process varies according to the vendor. In order to test the suspicion that quality depends on the vendor, the following sample data on the number of acceptable and unacceptable components were collected from shipments received from their three major component vendors:

	Items rejected	Items acceptable	
Vendor 1	12 (11.2)	112 (112.8)	124
Vendor 2	8 (7.9)	80 (80.1)	88
Vendor 3	12 (12.9)	129 (128.1)	141
	32	321	$n = 353$

Run a test at  $\alpha = 0.01$  to see if quality of shipment depends on the source of the shipment. (Show all elements of the testing procedure, including the  $p$ -value.) <20>

$H_0$ : quality and source are independent

$H_1$ : " " " " " " dependent

$$TS: \chi^2 = \sum_{i=1}^k \frac{(o_i - e_i)^2}{e_i} = \frac{(12 - 11.2)^2}{11.2} + \dots + \frac{(129 - 128.1)^2}{128.1} = \underline{0.133} \quad (df = 2)$$

$$CR: \text{reject } H_0 \text{ if } \chi^2 > 9.21$$

Fail to reject  $H_0$ . There is not enough evidence to show that quality is dependent on source of shipment.

$$p\text{-value} = P(\chi^2 > 0.133) \approx \underline{.99} \quad (\text{from } \chi^2 \text{ table})$$