

A unified computational modeling approach to decision making

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Abstract

For centuries, theorists have focused almost exclusively on the use of additive utility representations in attempts to describe decision behavior. This basic framework has constantly been modified in order to account for challenging empirical results. However, each revision to the basic theory has in turn consistently been confronted with conflicting evidence. Here, we summarize an alternative view focusing on the decision-making process. By employing computational models that offer a different—and arguably superior—level of analysis, we provide a more comprehensive account of human decision behavior. A survey of applications and discussion of parameter interpretation and estimation is also included.

Utility modeling

One can tentatively extract basic principles that seem to describe decision-making behavior. Some sort of affective value, or *utility*, is (at least theoretically) attached to possible outcomes or events. In cases of probabilistic or uncertain outcomes, degrees of belief or *decision weights* determine the relative perceived impact of different dimensions or potential outcomes. Somehow, these sources of information are presumably integrated to produce a preference for each option under consideration. Finally, a decision rule states which option to select, resulting in a corresponding action.

The calculus of probability and logic suggests that calculation of expected value provides a normative means to enact the decision program outlined above. By assigning values to possible outcomes, weighting each outcome by its likelihood of occurrence, summing these products to compute an *expected value*, and selecting so as to maximize expected value, one could adhere to an optimal decision policy. Descriptively, however, this “rational” account of decision making is subject to empirical verification, and has been overwhelmingly refuted. As a result, theorists have tried to identify and remedy descriptive inaccuracies in one or more of the constituent elements.

Pioneers in decision research suggested that perhaps the failure of expected value as a descriptive model resulted from subjective utility or evaluation $V(x)$ of values x (von Neumann & Morgenstern, 1944). This line of thought gave rise to *expected utility* models of decision making. However, this explanation still did not account for certain trends in behavior. As a result, theorists next focused on the

possibility that event probabilities p were also somehow transformed into a subjective belief or weight, $w(p)$. These two assumptions, concerning subjective assessment of value and belief, give rise to the general subjective expected utility form for an option X with outcomes x_i occurring with probability p_i (Savage, 1954):

$$U(X) = \sum w(p_i)V(x_i) \quad (1)$$

Subsequent empirical examination has placed an increasing number of constraints on the feasible functional forms for both the *value function*, $V(x)$, and the *weighting function*, $w(p)$. This has in turn led to increasingly complex algebraic equations that attempt to describe decision outcomes. Currently, *rank-dependent utility* theories, a class including prospect theory (Kahneman & Tversky, 1979), seem the most promising in terms of ability to account for empirical results. However, even this family of models is limited in terms of explanatory power and predictive scope (e.g., see Rieskamp, Busemeyer, and Mellers, 2005, for a review).

The goal of this paper is to summarize an alternative approach that enjoys advantages over even the most popular theories cast in the traditional utility framework. First, we introduce a specific computational modeling approach to decision making including treatment of each of the decision components mentioned above. We contrast this approach with the utility framework to highlight its strengths and gains in explanatory power. Then, we offer a brief survey from among the extensive successful applications of the basic model. We conclude with a discussion of important issues such as parameter estimation and interpretation.

Computational modeling via Markov processes

Our approach to modeling decision behavior relies on a formal mathematical model of the deliberation process. Specifically, we use Markov processes to model sequential sampling of information and accumulation of the sampled information over time. At this point, we have developed specific models for each of the component processes mentioned earlier. Before introducing these, it will be helpful to provide a brief introduction to computational modeling using Markov methods.

Markov models describe the state of a system $S(t)$ at time t , and are specified by defining some set of k possible states

for a system, $\mathbf{s} = \{s_1, \dots, s_k\}$; a probability distribution over initial states of the system, $\mathbf{z} = \{z_1, \dots, z_k\}$, where $z_i = \Pr[S(0) = s_i]$; valid transitions among states, and valid terminal (or absorbing) states¹. The latter two concepts are formalized respectively in a transition matrix \mathbf{Q} , where $q_{i,j}$ gives the probability of transiting from state i to state j , and an absorption vector \mathbf{a} , where a_i gives the probability of the system terminating when in state i . A schematic illustration of a simple Markov model is shown in Figure 1. It would be quite ambitious to make specific predictions about the state of the system at any given point in time, $\Pr[S(t) = s_i]$, for $i = 1, \dots, k$; more interesting and applicable for our purposes is to derive predictions about the probability distribution over terminal states of the system \mathbf{P} , which is given by (see Diederich & Busemeyer, 2003):

$$\mathbf{P} = \mathbf{z}' [\mathbf{I} - \mathbf{Q}]^{-1} \mathbf{a} \quad (2)$$

We can also easily compute the distribution \mathbf{T} of mean times to reach each terminal state $E[t^* | S(t^*) = s_i]$:

$$\mathbf{T} = (\mathbf{z}' [\mathbf{I} - \mathbf{Q}]^{-2} \mathbf{a}) / \mathbf{P} \quad (3)$$

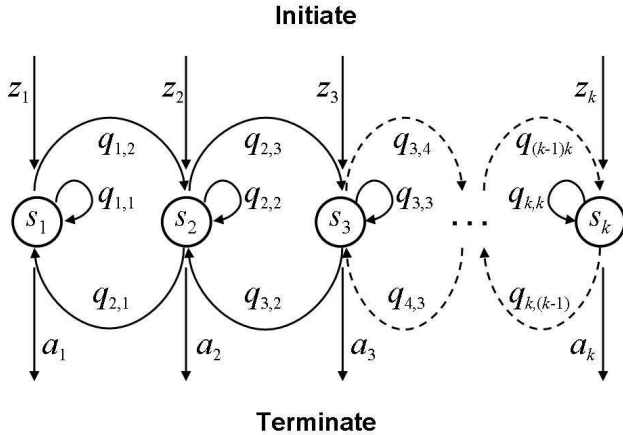


Figure 1: Generic Markov model representation

Information integration (preference accumulation)

The first application of the Markov model to decision making considered here is the *decision field theory* of Busemeyer and Townsend (1993). Decision field theory (DFT) specifies preference states and formalizes deliberation as transitions among states of differential preference for each of n options in a choice set. The actual process modeled is the integration and accumulation of sampled information over time. The resulting predictions are the probability of terminating in states which correspond to sufficient preference for selecting (choosing) one of the n options. In order to facilitate understanding and application

of the model, we ground the model introduction with a concrete example. We will use a decision task that is ubiquitous in experimental settings: choosing from among n probabilistic options, such as gambles, where option X is defined by outcomes x_i occurring with probability p_i . Following Busemeyer and Townsend (1993), we introduce the case of $n = 2$ for simplicity, but extensions to the multialternative case are provided by Roe, Busemeyer, and Townsend (2001).

Formally, define states s_1 and s_k as states of preference that warrant choice of the first (X_1) or second (X_2) option, respectively; the remaining states in \mathbf{s} are intermediate states of preference between these two extremes. We thus define $\mathbf{a} = [1 \ 0 \ \dots \ 0 \ 1]$ because only these states produce an overt choice (terminate deliberation). At the beginning of a decision, prior to any consideration of the options X_1 and X_2 , one may be in any intermediate state of preference, defined by the probabilities in $\mathbf{z} = [0 \ z_2 \ \dots \ z_{k-1} \ 0]$, which must sum to one. Consideration of information at each moment in the decision task results in either an increment in preference for X_1 , $S(t+1 | S(t) = s_i) = s_{i-1}$, or an increment in preference for X_2 , $S(t+1 | S(t) = s_i) = s_{i+1}$, for $i \neq 1, k$. The probability of each of these (exhaustive and mutually exclusive) events are precisely what we define in the transition matrix \mathbf{Q} : the former event given by $q_{i,i-1}$, and the latter given by $q_{i,i+1}$. Note that, as formalized here, other transitions are not valid, and the system cannot consecutively stay in the same state (i.e., $q_{i,j} = 0$, for $j \neq i-1, i+1$).

The crux of deliberation is sampling information about the options X_1 and X_2 over time that results in these momentary transitions. We therefore must define how the properties of X_1 and X_2 result in the transition probabilities in \mathbf{Q} . We assume that at each moment in time, some information x_i about each option is retrieved. This momentary attention to some specific attribute of each option produces a transient evaluation of, or *valence* for, each option $V(X_1)$ and $V(X_2)$. These momentary valences are compared to produce a difference value, $V = V(X_1) - V(X_2)$. Attention to different outcomes x_i produce different valences, and we can therefore define a sampling distribution of possible difference values, $V \sim (\mu, \sigma)$. Then a simple relationship exists between the parameters of this distribution and the elements of \mathbf{Q} ; if we define $d = \mu/\sigma$ as a measure of discriminability, then:

$$\begin{aligned} q_{i,i+1} &= \frac{1}{2} + \frac{1}{2} d \\ q_{i,i-1} &= \frac{1}{2} - \frac{1}{2} d \end{aligned} \quad (4)$$

In terms of the components of decision making outlined earlier, DFT describes the integration of information. It does so by specifying accumulation of preference resulting from sequential sampling, rather than instantaneous weighted summation implied by utility models. However, the model itself does not make specific claims about the evaluation and weighting components of the decision process. These elements correspond to the valences, V , and attention probabilities, $\Pr[V(t) = V_i]$, used to derive d . Extensions of the DFT framework have specified these elements using similar mathematical methods.

¹ Note that in the current paper we are dealing specifically with a discrete approximation to a continuous Markov process, assuming unitary time steps and discrete states.

Evaluation and weighting

The definition of the mean and variance of a random variable, such as $V(t)$, are given respectively by:

$$\mu = E[V(t)] = \sum V_i \cdot \Pr[V(t) = V_i] \quad (5)$$

$$\sigma^2 = \sum [V_i - \mu]^2 \cdot \Pr[V(t) = V_i] \quad (6)$$

Thus, if we can formalize the exact values V_i and the associated probabilities we can completely specify the DFT model. Busemeyer, Townsend, and Stout (2002) give a description of the former, and Johnson and Busemeyer (2005) give a model for the latter.

Busemeyer, et al (2002) have provided a specification of the evaluations associated with the attributes of choice options. They model these evaluations as the consequence of motivational dynamics in the level of need versus satiety for specific outcomes. The function of this system corresponds to the value function in utility models that use stated values as a basis for deriving subjective values. The model of Busemeyer, et al (2002) achieves this mapping by assuming that the objective values are weighted by a current need state, which is determined by the difference between an ideal or desired level of each outcome x_i and the current level of that outcome. For example, the greater the discrepancy between a current and desired level of an attribute, the greater perceived magnitude of that attribute. Over time, newly-acquired outcomes update the current attainment, which in turn updates the need state, which then alters current evaluations.

Johnson and Busemeyer (2005b) use a Markov model to describe how the sampling over these evaluations takes place. Specifically, they define a Markov model where the states correspond to momentary attention to each evaluation. The output of this system is selection of an evaluation $V(t)$ to contribute to preference accumulation. Practically, the model produces the probabilities $\Pr[V(t) = V_i]$, or *attention weights*. In other words, this model controls exactly which outcome x_i determines the momentary valence for each option at each point in time. So, whereas DFT describes transitions among preference states, Johnson and Busemeyer (2005b) show how a Markov model can detail the transitions in attention among evaluations that drive this evolution of preference. This model is similar in purpose to the weighting functions in utility models that transform objective probabilities into decision weights.

Formalized as a Markov model—distinct from the DFT model—it specifies initial attention to each outcome at each moment in \mathbf{z} . The model further assumes that the absorption probabilities are simply equal to the objective outcome probabilities, $a_i = p_i$. The model also allows for “dwelling” in the current state, or focusing on a given outcome for more than one moment, which is controlled by a parameter $0 < \beta < 1$. Due to mutually exclusive transitions, this suggests $q_{i,i} = (1 - a_i)\beta$ for all i . The transitions to other states, or attention to other outcomes, occurs with sum probability $(1 - a_i)(1 - \beta)$, which is then apportioned among the remaining elements $q_{i,j}$ for $j \neq i$. Using Equation 2, the model produces output probabilities that are interpreted as attention weights in the

choice model. That is, the weights here are attached to the evaluations in the Busemeyer, et al. (2002) model, allowing computation of μ and σ via Equations 5 and 6, the ratio of which in turn becomes d for determining the transition probabilities of the choice model via Equation 4.

Decision and response

Up to this point, we have given an overview of a cohesive mathematical description of outcome evaluation, outcome weighting, and the process that uses these elements to accumulate preference during deliberation. In this section, we complete the computational framework by specifying the decision rule and response selection mechanisms of decision making. The decision rule is guided by the transition into a terminal preference state, s_1 or s_k . Conceptually, if deliberation is described as the evolution of relative preference to options over time, then the decision rule can be thought of as a *threshold* level of preference required to stop deliberating and make a choice. This threshold can be adjusted by moderating the number of intermediate states—for example, increasing k will result in a stronger required level of preference which will amplify choice probabilities and increase deliberation time.

In some situations, the required response of a decision is not about selecting from among a given set of options, but assigning an appropriate value to a single option (e.g., pricing). In these instances, DFT as a model of choice cannot be applied directly. However, Johnson and Busemeyer (2005a) have formulated a third Markov model that uses the outputs of DFT as inputs in order to generate reported values. In this context, the states of the system become consideration of candidate values, and the output is the probability of reporting each value (e.g., assigning one of many possible prices). The initial distribution is then the probability of first considering each candidate value. Allowing terminal responses from any state (i.e., reporting any candidate value) suggests all nonzero elements in the absorption vector. The innovation of this model is that it recruits the DFT choice model to determine transitions among candidates, as follows.

The transition probabilities of the value response model are the output probabilities of the DFT choice model applied to the choice pair of the current candidate and the target option. Specifically, the probability that the comparison of candidate C_i and the target option produces terminal preference (i.e. enters state s_k) for C_i —suggesting C_i is too high—becomes the probability of transition to candidate $C_j < C_i$. Likewise, the probability of terminal preference (state s_1) for the target option determines the probability of transition to candidate $C_j > C_i$. The probability that C_i is a “good” value for the target option dictates the absorption probability for that value, a_i . This is determined by additionally allowing for absorption in the DFT choice model from the intermediate state where the difference value $V = 0$ (see Johnson & Busemeyer, 2005a, for details).

Modeling summary

We have shown how the use of a Markov architecture can be implemented at various stages of the decision making

process. In combination, the models reviewed above provide a complete computational model of decision behavior. Note that there is also opportunity for modifying some of the assumptions of the various models. For example, additional dependencies could be included, or valid transitions could be redefined. In this sense, the collection of models presented here can also be thought of as more of a modeling framework than a specific claim about the details of any given component. One admirable quality of this approach is the use of a single mechanism, albeit operating on various levels, to describe the components of decision making. Validation of this mechanism, then, becomes concurrent support for each of the component models. Table 1 shows how the different component models are mapped into a common (Markov process) architecture as depicted in Figure 1.

Comparison to utility models The key property that sets the computational approach apart from utility models is acknowledgement of the decision process, rather than solely the decision outcome. Where utility models may be able to specify a static preference ordering among a set of options, DFT postulates a specific model of the dynamic deliberation process that (effectively) produces this ordering.

Furthermore, DFT specifies preference strength and appreciates human variability via probabilistic outcome predictions, rather than the deterministic and binary consequences implied by the strict decision rule of utility maximization. Computational models, with their attention to the dynamics of the decision process, also naturally make predictions regarding deliberation time. This allows theory verification that is not possible with static, outcome-oriented utility equations that fail to specify how choices relate to deliberation time.

One benefit from using the computational approach outlined above is that it can, in fact, also emulate expected utility models. In the limit, as the decision threshold increases, the predictions of the Markov model coincide with those of expected utility, assuming the proper definition of the parameters of V (using Equation 1). Furthermore, lowering the threshold can be construed as a move to more “heuristic” processing, allowing the Markov model to mimic predictions from simpler models as well (cf. Lee & Cummins, 2004). In this sense, the Markov model is a more general model that subsumes other approaches as special cases.

Markov component	Choice model (DFT)	Evaluation model	Weighting model	Value response model
	<i>Busemeyer & Townsend (1993)</i>	<i>Busemeyer, Townsend, & Stout (2002)</i>	<i>Johnson & Busemeyer (2005b)</i>	<i>Johnson & Busemeyer (2005a)</i>
Initial state distribution	Initial relative preference across options	Current need state (differences in attained and desired levels)	Probability of initial attention to each outcome	Probability of first considering each candidate value
System states	States of relative preference for each option	Motivational tendencies towards each outcome	Attention to each outcome	Currently considered value
Transition matrix	Probability of increasing preference for each option	Changes in need state due to changes in attainment	Probability of shifting attention to each other outcome or dwelling on current outcome	Probability of considering each subsequent candidate value
Driving force	Momentary attention to specific evaluations	Evaluation of objective values relative to current need state	Shifting attention to produce momentary focus	Comparison of candidate values to target option (via choice model)
Absorption states	Correspond to sufficient (threshold) preference for an option	Correspond to current levels of motivation towards each outcome	Probability of incorporating associated outcome into preference state	Probability of reporting each candidate value
Output	Choice of option	Motivational values (evaluations) feed into choice model	Decision weights feed into choice model	Reported values

Table 1: Markov models of decision making components

Applications

We have provided a survey of a comprehensive computational approach to modeling decision making. This approach offers benefits over algebraic utility models in terms of the scope of predictions that can be made. More importantly, this approach should provide a good account of empirical data on human decision making. It is one thing to be able to make novel predictions, but it is still imperative to determine how accurate these predictions are. The models reviewed above have been applied to a number of situations that have challenged traditional utility models of decision making. Our goal here is not to provide an exhaustive review of these applications. However, by summarizing some of the successes of the computational approach, one can see that it is a viable and superior alternative to utility models in capturing many robust empirical trends.

First, we consider some “basic” properties that seem to characterize human decision behavior. Many of these properties serve as a sort of collective litmus test for judging models of preferential decision making (see Rieskamp, Busemeyer, and Mellers, 2005). For example, assume that $\Pr[\text{Choose A} \mid \{A, B\}] > 0.50 = A^*$ and $\Pr[\text{Choose B} \mid \{B, C\}] > 0.50 = B^*$. One basic property, *strong stochastic transitivity* (a probabilistic extension of transitivity), requires $\Pr[\text{Choose A} \mid \{A, C\}] > \max[A^*, B^*]$. Empirically this property is violated, but DFT predicts these violations (Busemeyer & Townsend, 1993). A second property, *stochastic dominance*, occurs when the cumulative density function for one option wholly exceeds another option: if $\Pr[x > x_i \mid A] > \Pr[x > x_i \mid B]$ for all x_i then A stochastically dominates B. In experimental settings where participants choose (nontransparently) stochastically dominated options, the weighting model produces weights that generate these violations (Johnson & Busemeyer, 2005b).

The weighting model also correctly predicts certain changes in preference across situations that should be mathematically equivalent according to standard utility models (see Johnson & Busemeyer, 2005b, for details regarding the following examples). Consider “event-splitting” effects, where preferences change when an outcome is decomposed into equivalent constituent outcomes (e.g., a 10% chance of winning \$20 is construed as a 5% chance of winning \$20 and another 5% chance of winning \$20). Even simply adding a common outcome to two options (common consequence effects) or multiplying outcomes of two options by a common value (common ratio effect) can induce changes in preference orderings. Again, these phenomena are emergent behaviors of the computational weighting model, but cannot be explained by traditional utility models.

An abundance of research has also shown different context effects on decision making. Broadly speaking, this means that changes in the context of the decision situation can result in changes in preferences. Simple manipulations of context are achieved by adding options to a choice set and measuring choice probabilities. Depending on the attributes of the additional option, preferences among the first two options are inconsistent. For example, assume two options A and B are equally preferred in a binary choice

task, to which a third option C is then added. If one of the original options (A) transparently dominates the additional option (C), then this original option (A) is preferred in the ternary choice set. However, if the additional option (C) is similar to, but not dominated by, the same original option (A), then the alternative option (B) is preferred in the ternary set.

Note that inconsistencies in this case are not only that one option is preferred in the ternary set (although it was not in the binary set), but also the preferred option changes based on the properties of the additional option. Both of these results contradict entire classes of algebraic utility models. Roe et al. (2001) show how DFT accounts for these results through competitive feedback incorporated in the multialternative extension of the valence difference, V . Other empirically-observed context effects that have been explained using the computational models outlined here include loss aversion (see Johnson & Busemeyer, 2005b) and endowment effects (see Busemeyer & Johnson, 2004).

Beyond characteristics of the stimulus set, decades of research have shown how the task itself (i.e. response method) can also produce inconsistencies in preference orderings. Johnson and Busemeyer (2005a) review various instances of how changes in the response mode, rather than characteristics of the task itself, can produce changes in preference orders. For example, given a pair of gambles (with certain properties), people may choose one gamble in a forced choice, but assign a higher price to another. Also, people may assign a higher buying price to one gamble, but a higher selling price to another. Johnson and Busemeyer (2005a) not only explain why utility models (and other approaches) cannot account for a wide range of these phenomena, but show how the computational response model above can.

The models described herein can also make novel predictions about the dynamics in decision behavior. For example, DFT explains the robust tradeoff between deliberation time and accuracy (Busemeyer & Townsend, 1993). Some of the effects above are strengthened or attenuated with increased deliberation time as well, another application of the models (e.g., Roe, et al, 2001; Diederich, 2003). These models can also help understand the long-term dynamics in decision behavior. Johnson and Busemeyer (2005c) have incorporated feedback from one trial into model parameters for subsequent trials to explain effects such as the emergence of routine behavior. Furthermore, Johnson and Busemeyer (2001) detail how DFT can account for dynamic inconsistency in choice behavior (i.e., changes in preference as a function of delay between the point of decision and outcome realization). Busemeyer, et al. (2002) apply the computational framework to explain motivational dynamics driven by need and satiation that have been found experimentally.

Parameter estimation and interpretation

One may criticize the computational models presented here on a number of grounds. First, one may argue that these models are too complex for practical application. A related conjecture is that the models are complex in an information-theoretic sense, and that is the reason they are able to make

such extensive predictions. In reality, these models contain relatively few free parameters. Furthermore, in the applications described above, these parameters were usually held constant across applications, so that a single set of parameter values produced multiple phenomena. It is also important to note that the models are not so flexible that they are able to reproduce any data trend. For example, Johnson and Busemeyer (2005a) show that the value response model predicts preference reversals only in those situations (i.e., using the appropriate stimuli) where such effects have been obtained experimentally.

Another primary advantage of the computational approach outlined here is the potential to estimate model parameters from independent data, and/or from the experimental stimuli. Busemeyer and Townsned (1993) provide one such method for the DFT choice model. Raab and Johnson (2004) preset DFT model parameters on the basis of an independent personality inventory in order to explain decision behavior in an applied task. Furthermore, with the advent of increasingly precise process-tracing techniques and accompanying analytic procedures, it may be possible to preset model parameters based off other measurements. For example, it would be worthwhile to examine the extent to which eye-tracking data could be useful in verifying (or setting *a priori*) the relative attention to each outcome in a decision task.

A final advantage of the computational approach is the interpretability of the model parameters and psychological plausibility of hypothesized processes. Rather than attaching *ad hoc* interpretations to algebraic exponents and coefficients, the computational model suggests straightforward interpretation of the included parameters. Thresholds in the choice model can represent impulsive or methodical deliberation; initial state distributions can reflect biases brought into a decision situation; the dwelling parameter in the weighting model indicates the tendency to ponder; etc. Finally, it should be noted that the gradual accumulation-to-threshold mechanism posited here has indeed been supported by neural recording from primates involved in simple decision tasks (Smith & Ratcliff, 2004).

Summary and conclusion

We summarized here an approach to modeling decision making that departs from the theoretical norm (expected utility). Rather than continuing attempts to repair utility models in light of mounting contradictory evidence, we have opted instead to adopt a different level of analysis. Specifically, we use mathematical (Markov) techniques to derive models of core processing components of decision making. These models function on distinct but connected levels to provide a comprehensive framework of decision making. This approach has accounted for an abundance of empirical trends that have challenged competing models

The key innovation of the models presented here is the attention to modeling the decision process, rather than just outcomes. This also enables the model to make predictions beyond the scope of algebraic utility models, such as predictions regarding information search, deliberation time, and response variability. Although further tests of the constituent models are necessary to confidently claim

superiority over competing models, the applications reviewed here indicate remarkable initial success.

References

- Busemeyer, J. R. & Johnson, J. G. (2004). Computational models of decision making. In D. Koehler & N. Harvey (Eds.) *Handbook of Judgment and Decision Making*. Blackwell Publishing Co.
- Busemeyer, J. R., & Townsend, J. T. (1993). Decision Field Theory: A dynamic cognition approach to decision making. *Psychological Review*, *100*, 432-459.
- Busemeyer, J. R., Townsend, J. T., & Stout, J. C. (2002) . Motivational underpinnings of utility in decision making: decision field theory analysis of deprivation and satiation. In S. Moore (Ed.) *Emotional Cognition*. Amsterdam: John Benjamins.
- Diederich, A. (2003). MDFT account of decision making under time pressure. *Psychonomic Bulletin & Review*, *10*, 157-166
- Diederich, A., & Busemeyer, J. R. (2003). Simple matrix methods for analyzing diffusion models of choice probability, choice response time and simple response time. *Journal of Mathematical Psychology*, *47*, 304-322.
- Johnson, J. G. & Busemeyer, J. R. (2005a). A dynamic, stochastic, computational model of preference reversal phenomena. *Psychological Review*, *112*, 841-861.
- Johnson, J. G. & Busemeyer, J. R. (2005b). A computational model of the decision weighting process. Manuscript in preparation.
- Johnson, J. G. & Busemeyer, J. R. (2005c). Rule-based decision field theory: A dynamic computational model of transitions among decision-making strategies. In T. Betsch & S. Haberstroh (Eds.), *The routines of decision making*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Johnson, J. G. & Busemeyer, J. R. (2001) Multiple stage decision making; The effect of planning horizon on dynamic consistency. *Theory and Decision*, *51*, 217-246.
- Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, *47*, 263-291.
- Lee, M. D., & Cummins, T. D. R. (2004). Evidence accumulation in decision making: Unifying the 'take the best' and 'rational' models. *Psychonomic Bulletin & Review*, *11*(2), 343-352.
- Raab, M. & Johnson, J. G. (2004). Individual differences of action-orientation for risk-taking in sports. *Research Quarterly for Exercise and Sport*, *75*(3), 326-336
- Rieskamp, J., Busemeyer, J. R., & Mellers, B. A. (2005) Extending the bounds of rationality : A review of research on preferential choice. *Journal of Economic Literature*.
- Roe, R. M., Busemeyer, J. R., & Townsend, J. T. (2001) Multi-alternative decision field theory: A dynamic connectionist model of decision making. *Psychological Review*, *108*, 370-392
- Savage, L.J. (1954): *The Foundations of Statistics*. New York, NY: Wiley.
- Smith, P. L., & Ratcliff, R. (2004). Psychology and neurobiology of simple decisions. *Trends in Neurosciences*, *27*(3), 161-168.